D 102522 (Pages : 11) Name.....

Reg. No.....

P.G. ENTRANCE EXAMINATION, APRIL 2024

MATHEMATICS

Time : Two Hours Maximum : 200 Marks

Answer **all** *the questions.*

Each question carries 4 *marks.*

1 *mark will be deducted for each wrong answer.*

Symbols :

$\mathbb{R}$: The set of all real numbers $\mathbb{C}$: The set of all complex numbers

$\mathbb{Z}$: The set of all integers $\mathbb{Q}$: The set of all rational numbers

det(A) : determinant of A.

1. Which one among the following is a possible order of a non-Abelian group ?

   (A) 131. (B) 169.

   (C) 120. (D) 127.

2. What is the GCD of 2261 and 1275 ?

   (A) 27. (B) 17.

   (C) 47. (D) 37.

3. What is value of the contour integral $\frac{1}{2\pi i}\int_{|z|=2}\frac{ze^z}{z-i}\,dz$?

   (A) $-2\pi e^i$. (B) ie^i.

   (C) $2\pi e^{2i}$. (D) $2\pi i e^{2i}$.

4. Consider the function $f(z)=\frac{1}{\sin\left(\frac{1}{z}\right)}$. What is the nature of singularity at $z = 0$?

   (A) Simple pole. (B) Isolated singular point.

   (C) Non isolated singular point. (D) A double pole.

5. Consider the function $f(z)=\frac{\sin z}{\left(z^2-1\right)^2}$. What is the order of the pole at $z_0 = 1$?

   (A) 1. (B) 2.

   (C) 3. (D) 4.

Turn over

6. What is the value of the integral $\int_{|z|=1} \frac{2}{z^2+4z+1}\,dz$?

(A) $\frac{-2}{\sqrt{3}}\pi i.$ (B) $2\pi i.$

(C) 0. (D) $\frac{2}{\sqrt{3}}\pi i.$

7. Consider the function $f(x,y,z)$ defined everywhere except at the origin. What is the gradient of the scalar field ?

(A) $\frac{x\hat{i}+y\hat{j}+z\hat{k}}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}.$ (B) $\frac{-x\hat{i}-y\hat{j}-z\hat{k}}{\left(x^2+y^2+z^2\right)^{\frac{3}{2}}}.$

(C) $\frac{-x\hat{i}-y\hat{j}-z\hat{k}}{\left(x^2+y^2+z^2\right)^{\frac{1}{2}}}.$ (D) $\frac{x\hat{i}-y\hat{j}+z\hat{k}}{\left(x^2+y^2+z^2\right)^{\frac{1}{2}}}.$

8. Which of the following is the parametrization of the cylinder $x^2+(y-3)^2=9, 0\le z\le 5$?

(A) $r(\theta,z)=(3\sin 2\theta)\hat{i}+(6\sin^2\theta)\hat{j}+z\hat{k}, 0\le\theta\le\pi.$

(B) $r(\theta,z)=(3\cos 2\theta)\hat{i}+(6\cos^2\theta)\hat{j}+z\hat{k}, 0\le\theta\le\pi.$

(C) $r(\theta,z)=(3\cos 2\theta)\hat{i}-(6\cos^2\theta)\hat{j}-z\hat{k}, 0\le\theta\le 2\pi.$

(D) $r(\theta,z)=(3\sin 2\theta)\hat{i}-(6\sin^2\theta)\hat{j}-z\hat{k}, 0\le\theta\le 2\pi.$

9. What is the circulation $\oint_C xydy-y^2dx$ where C is the square cut from the first quadrant by the lines $x = 1$ and $y = 1$?

(A) $\frac{5}{2}.$ (B) $\frac{7}{3}.$

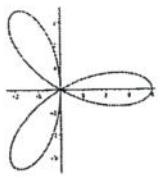
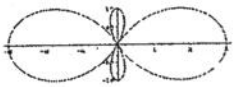
(C) $\frac{11}{3}.$ (D) $\frac{3}{2}.$

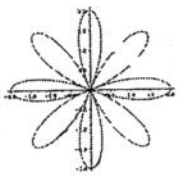
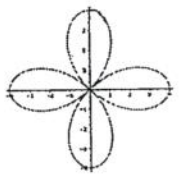
10. Consider the spherical co-ordinates $x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\phi$. What is the Jacobian of the transformation from Cartesian $\rho\phi\theta$-space to Cartesian xyz-space ?

(A) $\rho^2 \cos\phi$. (B) $\rho \sin^2\phi$.

(C) $\rho^2 \sin\phi$. (D) $\rho \sin\phi$.

11. Which one among the following represents the curve $r = 4\cos(2\theta), 0 \le \theta \le 2\pi$?

(A)  (B) 

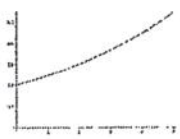
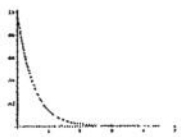
(C)  (D) 

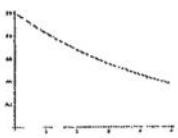
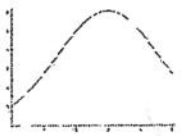
12. Suppose $\vec{a} = 2\hat{i} + \hat{j} + \hat{k}, \vec{b} = -4\hat{i} + 3\hat{j} + \hat{k}$. What is $\vec{a} \times \vec{b}$?

(A) $2\hat{i} + 6\hat{j} - 10\hat{k}$. (B) $-2\hat{i} - 6\hat{j} + 10\hat{k}$.

(C) $-2\hat{i} + 6\hat{j} + 10\hat{k}$. (D) $2\hat{i} - 6\hat{j} + 10\hat{k}$.

13. Identify the solution of the initial value problem $\frac{dy}{dx} = 0.4y(3-x), y(0) = 1$ on the interval $[0, 5]$:

(A)  (B) 

(C)  (D) 

14. Which one among the following is the row echelon form of the matrix $A = \begin{pmatrix} 21 & 12 & 3 \\ 41 & 12 & 11 \\ 8 & -3 & -2 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$.

(B) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(C) $\begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$.

(D) $\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}$.

15. Which one among the following is the inverse of the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$?

(A) $\begin{pmatrix} -40 & -16 & 9 \\ -12 & 5 & -3 \\ 5 & -2 & 1 \end{pmatrix}$.

(B) $\begin{pmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{pmatrix}$.

(C) $\begin{pmatrix} -40 & -16 & 9 \\ -12 & 15 & -3 \\ 5 & -2 & 11 \end{pmatrix}$.

(D) $\begin{pmatrix} 40 & -16 & 9 \\ 12 & 5 & -3 \\ 5 & 12 & 11 \end{pmatrix}$.

16. Which one among the following is the characteristic polynomial of the matrix $A = \begin{pmatrix} 1 & 7 & -3 \\ 12 & -5 & 6 \\ 4 & 2 & -8 \end{pmatrix}$?

(A) $x^3 - 12x^2 + 57x + 736$.

(B) $x^3 + 21x^2 - 157x - 786$.

(C) $x^3 + 121x^2 - 17x - 796$.

(D) $x^3 + 12x^2 - 57x - 736$.

17. For the sequence $f_n : [0,1] \to \mathbb{R}$ defined by $f_n(x) = \begin{cases} 2n^2x & \text{if } 0 \le x \le \frac{1}{2n} \\ 2n^2\left(\frac{1}{n} - x\right) & \text{if } 0 \le x \le \frac{1}{2n} \\ 0 & \text{if } \frac{1}{n} \le x \le 1. \end{cases}$ Which of the following is correct ?

(A) f_n diverges.

(B) f_n is pointwise convergent but not uniformly convergent.

(C) f_n is uniformly convergent.

(D) $f_n(x)$ is converging pointwise to π^2.

18. Consider the sequence of functions $f_n : \mathbb{R} \to \mathbb{R}$ defined by $f_n(x) = \frac{x}{1+nx^2}$. Which one of the following statements is correct ?

(A) f_n converges uniformly to $\pi + e$.

(B) $f_n' \to g$ pointwise, where $g = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0. \end{cases}$

(C) $f_n' \to g$ uniformly, where $g = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{if } x \neq 0. \end{cases}$

(D) $f_n' \to g$ uniformly, where $g = \begin{cases} \pi & \text{if } x = 0 \\ e & \text{if } x \neq 0. \end{cases}$

19. Consider the function $f(x) = e^x + 4x - 2$. Which of the following statements is correct ?

(A) $f(x)$ doesn't have any zero in the interval $\left(0, \frac{1}{2}\right)$.

(B) $f(x)$ has a zero in the interval $(-1, 0)$

(C) $f(x)$ has a zero in the interval $\left(0, \frac{1}{2}\right)$.

(D) $f(x)$ has a zero in the interval $\left(\frac{1}{2}, \frac{3}{2}\right)$.

20. What is the average value of $f(x,y) = x\cos(xy)$ over the rectangle $0 \le x \le \pi, 0 \le y \le 1$?

(A) $\frac{2}{\pi}$.

(B) $\frac{2}{3\pi}$.

(C) $\frac{3}{\pi}$.

(D) $\frac{5}{\pi}$.

21. Which one among the followng is the tangent plane to the surface $z = x\cos y - ye^x$ at the origin ?

(A) $x + y + z = 0$.

(B) $x - y - z = 0$.

(C) $x + y - z = 0$.

(D) $-x + y - z = 0$.

22. What is the linearization of the function $f(x,y,z) = x^2 - xy + 3\sin z$ at the point (2, 1, 0) ?

(A) $3x + 2y + 3z - 2$.

(B) $-3x + 4y - 3z - 2$.

(C) $3x - 4y + 6z - 2$.

(D) $3x - 2y + 3z - 2$.

23. Consider the function $g : [0, 1] \to \mathbb{R}$ defined by $g(x) := \begin{cases} \sin\left(\frac{1}{x}\right) & \text{for } 0 \le x < 1 \\ 0 & \text{for } x = 0. \end{cases}$ Which of the following statements is correct ?

(A) g is not differentiable at any point in (0, 1).

(B) g is differentiable at every point in (0, 1).

(C) g is not differentiable at a countably many points inside (0, 1).

(D) g is differentiable and the derivative is Riemann integrable on [0, 1].

24. Consider the polynomials $f(x) = 3x^4 + x^3 + 2x^2 + 1$ and $g(x) = x^2 + 4x + 2$ in $\mathbb{Z}_5[x]$. What is the remainder when $f(x)$ is divided with $g(x)$?

(A) $3x - 2$.

(B) $3x^2 - 2x + 1$.

(C) $-4x^3 + 3$.

(D) $2x + 1$.

25. What is the slope of the line $8x + 5y = 20$?

(A) $\frac{20}{8}$.

(B) $-\frac{8}{5}$.

(C) $\frac{5}{8}$.

(D) $-\frac{1}{5}$.

26. Which of the following is not an integral domain ?

(A) The ring of Gaussian integers $\mathbb{Z}[i] = \{a + bi; a, b \in \mathbb{Z}\}$.

(B) The ring $\mathbb{Z}_p$ of integers modulo a prime p.

(C) The ring $M_2(\mathbb{Z})$ of 2×2 matrices over the integers.

(D) The ring $\mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2}; a, b \in \mathbb{Z}\}$.

27. What is the radius of convergence of the series expansion of $f(z) = \frac{2z+3}{z+1}$ as powers of $(z - 1)$?

(A) 1. (B) ∞.

(C) 2 (D) $\frac{3}{2}$.

28. Find the value of the integral $\int_{|z|=r} \Re(z)dz$? ($\Re(z)$ is the real part of z) :

(A) $2\pi r$. (B) $i\pi r^2$.

(C) $2\pi i r^2$. (D) $4\pi i r^2$.

29. If X is a binomially distributed random variable with 6 trials and a probability of success is $\frac{1}{4}$ at each attempt, what is the probability of at least one success ?

(A) $\frac{3217}{4026}$. (B) $\frac{3456}{4096}$.

(C) $\frac{3367}{4096}$. (D) $\frac{2456}{4026}$.

30. The mean number of bacteria per millilitre of a liquid is known to be 6. What is the probability that in 1 ml of the liquid, there will be 6 bacteria ?

(A) $e^6 \times \frac{6^6}{6!}$. (B) $e^6 \times \frac{6^3}{6!}$.

(C) $e^{-6} \times \frac{6^6}{3!}$. (D) $e^{-6} \times \frac{6^6}{6!}$.

31. Which one among the following statement is not correct ?

(A) $f(x) = 3x + 7$ is uniformly continuous on $\mathbb{R}$.

(B) $f(x) = x^2$ is uniformly continuous on $[0, 4]$.

(C) $f(x) = x^2$ is uniformly continuous on $(0, \infty)$.

(D) $f(x) = \dfrac{1}{x}$ is not uniformly continuous on $[0, \infty]$.

32. Find the point at which the function $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$ has a local extremum :

(A) $(-2, 2)$. (B) $(3, -2)$.

(C) $(-2, -2)$. (D) $(2, -3)$.

33. What is the volume of the prism whose base is the triangle in the xy-plane bounded by the x-axis and the lines $y = x$ and $y = 1$ and whose top lies in the plane $f(x,y) = 3 - x - y$?

(A) 2 units. (B) 11 units.

(C) 1 unit. (D) 22 units.

34. Find the integrating factor for the differential equation $xy' - 2y = x^3 \sin x$:

(A) $\dfrac{1}{\log x}$. (B) $\dfrac{1}{x^2}$.

(C) $\dfrac{1}{\sin x}$. (D) $\dfrac{1}{\log x^2}$.

35. Which one among the following properties listed for the convolution of two functions is not correct ?

(A) $f(x) * g(x) = g(x) * f(x)$.

(B) $0 * f(x) = 0$.

(C) $1 * f(x) = f(x)$.

(D) $[\alpha f(x) + \beta g(x)] * h(x) = \alpha[f(x) * h(x)] + \beta[g(x) * h(x)]$.

36. Which of the following is a ring homomorphism ?

(A) The map $n \mapsto n^k$ defined on the set of integers $\mathbb{Z}$, where $k > 1$.

(B) $\phi_a : \mathbb{Z}[x] \to \mathbb{Z}$ defined by $\phi(p(x)) = p(a)$, where $a \in \mathbb{Z}$.

(C) $\phi : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n + 1$.

(D) The map $n \mapsto 2n$ defined on the set of integers $\mathbb{Z}$.

37. What is the order of the element (8, 56) in the group $\mathbb{Z}_{12} \times \mathbb{Z}_{60}$?

(A) 17.
(B) 25
(C) 35.
(D) 15.

38. Suppose the matrix $A = \begin{pmatrix} 3 & 1 \\ 5 & -2 \end{pmatrix}$ satisfies $A^3 = aA + bI$. What are the values for a and b ?

(A) $a = 11, b = 12$.
(B) $a = 12, b = 11$.
(C) $a = 21, b = 13$.
(D) $a = 13, b = 23$.

39. What is the limit of sequence of partial sums of the series $\sum_{n=0}^{\infty} 3^{2+n} 2^{1-3n}$?

(A) $\frac{124}{55}$.
(B) $\frac{142}{5}$.
(C) $\frac{144}{5}$.
(D) $\frac{242}{55}$.

40. What will be the nature of the singularity at $z = \pi$ of the functon $f(z) = \frac{1 + \cos(z)}{(z - \pi)^2}$?

(A) a double pole.
(B) an isolated essential singular point.
(C) a non-isolated essential singular point.
(D) a removal singular point.

41. What is the residue at the singular points of the function $f(z) = \frac{z^2}{z^3 - 8}$?

(A) $\frac{1}{3}$.
(B) $\frac{1}{2}$.
(C) $\frac{2}{3}$.
(D) $\frac{8}{3}$.

42. In how many ways can you choose a president, secretary and treasurer for a club from 12 candidates, if each one eligible for each position and no candidate can hold two positions ?

(A) 1230. (B) 1023.

(C) 2024. (D) 1320.

43. Which of the following is the inverse Laplace transform of $x(s) = \frac{s}{s^2a^2 + b^2}$?

(A) $\frac{1}{a^2}\sin\left(\frac{b}{a}\right)t.$ (B) $\frac{1}{a^2}\cos\left(\frac{a}{b}\right)t.$

(C) $\frac{1}{a^2}\cos\left(\frac{b}{a}\right)t.$ (D) $\frac{1}{a^2}\sin\left(\frac{a}{b}\right)t.$

44. What is the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x, -1 \le x \le 2$?

(A) $\frac{13}{72}.$ (B) $\frac{73}{21}.$

(C) $\frac{37}{12}.$ (D) $\frac{17}{12}.$

45. What is the length of the curve $y = \frac{4\sqrt{2}}{3}x^{\frac{3}{2}} - 1, 0 \le x \le 1$?

(A) $\frac{12}{7}.$ (B) $\frac{13}{6}.$

(C) $\frac{14}{8}.$ (D) $\frac{15}{9}.$

46. Let G be a finite group and Z(G) is its center. Which one among the following statements are incorrect ?

(A) If G/Z(G) is cyclic, then G is Abelian.

(B) If $|G| = 15$, then it is cyclic.

(C) If $|G/Z(G)| = 15$, then G is Abelian.

(D) If $|G/Z(G)| = 17$, then G is a non-Abelian group.

47. What is the derivative of x^2, $x > 0$?

(A) $x \times x^{\log x - 1}$.

(B) $x^x \left(\log x^x\right)$.

(C) $x^x (1 + \log x)$.

(D) $x^x \left(1 + \log x^x\right)$.

48. Suppose you deposit Rs. 621 in a bank account that pays 6 % interest compounded annually. How much money will you have 8 years later ?

(A) $621 \times e^{0.06}$.

(B) $621 \times e^{0.08}$.

(C) $621 \times e^{0.48}$.

(D) $621 \times e^{0.14}$.

49. What is the units digits of 43^{17} ?

(A) 2.

(B) 3.

(C) 7.

(D) 4.

50. What will be the remainder when 4137192658 is divided by 11 ?

(A) 7.

(B) 8.

(C) 5.

(D) 3.

(50 × 4 = 200 marks)